Randomly Guided Mesh Adaptive Direct Search Algorithm Applied for Optimal Design of Electric Machines based on FEA

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Abstract — Optimal design of electric machine based on FEM (Finite Element Method) calls for much longer computation time. In this paper, optimal design is implemented with randomly guided MADS (Mesh Adaptive Direct Search) and FEA (Finite Element Analysis) to compensate the excessive computation time. In addition, the proposed MADS coupled with FEA has been forwarded to Optimal design of Interior PM Synchronous Machine for Maximum Torque Per Ampere (MTPA). In particular, Randomly guided MADS has contributed to reducing the excessive computing time for the optimization process when compared with conventional MADS.

I. Introduction

Optimization algorithms applied to the optimal design of electric machines, which is characterized with the nonlinear magnetic saturation and has many local optima, have been focused on reducing computation time [1]-[3]. Thus, guided MADS has been implemented to compensate the excessive computation time to global optimum adaptively in the multi-modal problems.

This paper presents a randomized version of guided MADS. MADS is one of the direct search methods and generates random trial points to search the best local minima [4]. Whereas, the guided MADS modifies the poll points using the relationship of the previously computed trial points, which has contributed to reducing the computation time more effectively than MADS.

The proposed randomly guided MADS has been applied to the optimal design of IPMSM for Maximum Torque Per Ampere (MTPA) with the many local optima requiring the much longer computation time.

II. Guided Mesh Adaptive Direct Search (MADS)

MADS is a highly flexible local search method by blending the random selection rule in generalized pattern search (GPS) [5]. MADS is local search algorithm composed of the search step and the poll step. At the search step, MADS generates a number of trial points on the current mesh according to the fixed directions in a positive spanning set. At the poll step that follows, random points are generated to cover the whole current frame, i.e. a collection of the current unit meshes as components. It should be noted that the poll size parameter Δ_k^p should be equal to or larger than the mesh size parameter Δ_k^m for ensuring local convergence [6].

This paper proposes a new concept of heading direction in accordance with landscape shape for efficient local search, which is used as a reference direction for further randomly extended search. MADS is an iterative algorithm where at the k-th iteration a finite number of trial points are constructed from $D \subset R^n$, a fixed set of n_D directions scaled by a mesh size parameter $\Delta_k^m \in R_+$. In general, each basis direction $d_j \in D,\ j = 1, \cdots, n_D$ is set with coordinate directions which can be denoted in n-dimensional space as $\{e_i, -e_i\}, i = 1, \cdots, n$. In case of the smooth cost landscape around the incumbent search point, MADS will have a higher chance to find an improved point along the most promising direction with multiple attempts. In this paper, the promising direction named heading direction is estimated with the current cost values.

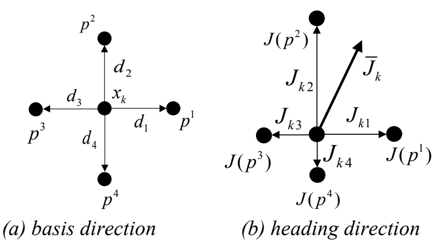
The heading direction at the k-th iteration is computed by a combination of the cost values of the current trial points. Figure 1(a) shows basis directions and the trial points $p_i, i = 1, \cdots, 4$ generated around x_k with a step length Δ_k^m in two dimensions. In Fig. 1(b), cost values are evaluated for each trial point and the heading direction is attained by a weighted sum of successful search directions as

$$\bar{J}_k = \frac{d_1 J_{k1} + d_2 J_{k2} + \cdots + d_{2n} J_{k2n}}{\sqrt{J_{k1}^2 + J_{k2}^2 + \cdots + J_{k2n}^2}} \tag{1}$$

where $J_{ki} = \left| \min\left(0, \dfrac{J(p^i) - J(x_k)}{\left|p^i - x_k\right|}\right) \right|, i = 1, \cdots, 2n$ is the cost strength of the i-th point. Then, $\bar{J}_k$ is normalized to be a heading direction. In the successful poll step, m random points are generated with reference to $\bar{J}_k$ and the best two trial points with the following relationship

$$d_i^e = \bar{J}_k + \alpha_1 d^{(1)} + \alpha_2 d^{(2)}, \quad i = 1, \cdots, m \tag{2}$$

where α_1 and α_2 are uniform random numbers between -0.5 and 0.5, and $d^{(1)}$ and $d^{(2)}$ denote the best and the second best trial points. The extended search with random trial points is carried out until no improved point is found.



(a) basis direction *(b) heading direction*

Fig. 1. Concept of estimating the heading direction with cost values

III. Design Charateristics of IPMSM

A. Modeling of IPMSM considering magnetic saturation

IPMSM has the distinguished rotor structure with PM buried interiorly, which causes the significant magnetic saturation. For reference, conventional d-q voltage and torque equations of IPMSM under the steady state operation can be represented as follows [7].

$$\begin{aligned} V_d &= R_s i_d - \omega_r L_q i_q \\ V_q &= R_s i_q + \omega_r L_d i_d + \omega_r \lambda_m \end{aligned} \tag{3}$$

$$T_e = \frac{3}{2}\frac{p}{2}\left\{\lambda_m i_q + (L_d - L_q) i_d i_q\right\} \tag{4}$$

The representative parameters of d-q flux linkages have been obtained numerically based of non-linear Finite Element Method at first, then are forwarded to the resultant torque equation which is formulated in Eq.(4), of which resultant torque is already validated by the directly obtained torque by Maxwell Stress Tensor.

B. Design objective of IPMSM: MTPA

This condition is useful for starting mode and starting torque is limited by allowable maximum current, not by voltage limit on account of low speed. Hence design objective for this condition is usually to be MTPA as follows using synthetic flux linkage (λ_d, λ_q). In this paper, MTPA is selected as objective function in this optimal design of IPMSM.

$$MTPA = MAX\left[\frac{3}{2}\frac{p}{2}(\lambda_d i_d - \lambda_q i_q)/I_{\max}\right] \tag{5}$$

C. Design variables and constraints

In this paper, the purposely built IPMSM(11.3kW, 1800rpm) has been applied for the optimal design. Under the fixed outline dimension maintaining the outer diameter(=90[mm]) and the axial length(=200[mm]), the design variables are selected as the side-PM length(X_1) and thickness(X_2), the horizontal-PM length(X_3) and thickness(X_4), angle between the side-PM and the horizontal-PM(X_5), as shown in fig2. The other dimensions, such as the number of poles(=10) and slots(=45), the residual flux density of PM (Br=1[T]), the air-gap length(=0.8[mm]), and so on, have been fixed. In addition, the design constraint is the quantity consumed PM (cross section of PM is under 84mm^2).

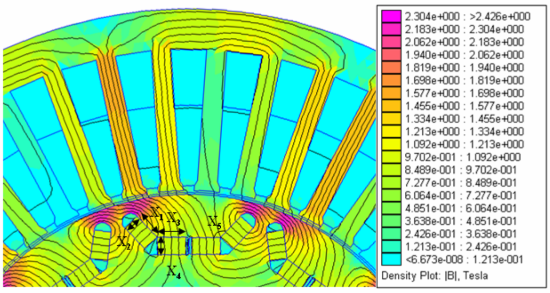


Fig. 2. Design variables for the purposely built IPMSM

IV. Numerical Validation Results

The well-known mathematical functions such as Branin function, is used to verify the effectiveness of the proposed randomly Guided MADS, of which are shown in Fig 3. The Brain function is a global optimization test-function having only two variables.

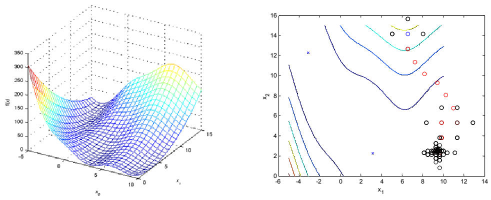


Fig. 3. Branin function and Result of convergence

The function has three equal-sized global optima, and has the following definition

$$f_{br}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10 \tag{6}$$

Where $-5<x_1<10$, $0<x_2<15$, the global optima equal $f(x_1, x_2)=0.397887$ are located as follows: $(-\pi,12.275)$, $(\pi,2.275)$, $(9.42478,2.475)$.

In terms of the function evaluation number, Randomly Guided MADS can generate the optimal design solution for IPMSM with fast convergence. Results of the Randomly Guided MADS compared with MADS and Optimal design for IPMSM will be minutely shown in Full-paper.

V. Conclusion

Through application of the proposed method to the optimal design of IPMSM requiring the huge computational time for FEA(Finite Element Analysis) and having many local optima, it will be shown that this method is very powerful to the multimodal optimization and appropriate for design of electric machine.

VI. References

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